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Technical Notes

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Hybrid Method for Jet Vane Thermal Analysis in Supersonic Nozzle Flow

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Nomenclature

i_s = stagnation enthalpy
k = specific heat ratio
M_1 = Mach number in a free-stream on a vane surface
Pr = Prandtl number
q''_0 = heat flux to a vane surface
Re = Reynolds number
r = recovery factor on a vane surface
St = Stanton number
T_0 = total temperature of a freestream on a vane surface
T_r = recovery temperature on a vane surface
T_w = wall temperature of a jet vane
T_∞ = static temperature of a freestream on a vane surface
u_∞ = freestream velocity on a vane surface
Δ_2 = enthalpy thickness of a thermal boundary layer
μ_∞ = viscosity of a freestream on a vane surface
ρ_∞ = density of a freestream on a vane surface

I. Introduction

NUMERICAL analysis based on finite difference method (FDM) has a weakness in the accurate calculation of convective heat transfer in a supersonic flow. Especially, if the flow is turbulent, the problem becomes more sophisticated because the accuracy of results depends on many factors such as turbulent model, grid fineness near a surface, and flow characteristics (cf. high accelerating or decelerating flow) on it. The computation time is increased considerably by the increased numerical grid resolution and the equations of the turbulent model. Also, the grid generation process needs careful and time-consuming work to produced grid structure sufficient for an accurate simulation of a boundary layer. Therefore, these preconditions for an accurate calculation could cause inefficiency of the total analytical process.

In this study, a hybrid method is used to calculate the convective heat transfer on a surface. Potential flow field around a solid surface is calculated by a computational fluid dynamics (CFD) solver and the heat transfer on a surface is analyzed separately with the solution of an integral form of thermal boundary layer equation. Since the 1960s, this integral form of boundary layer equation has been used by many investigators to derive semiempirical relations for the skin-friction coefficient or the heat-transfer coefficient from the experimental results. Practically, this analytic method showed results comparable to those of the experimental one even when it was applied to the convergent–divergent nozzle[1] or the diffuser[2] where accelerating or decelerating flow was predicted. Also, a relation was suggested to correct for the difference between the temperature and velocity profiles in a compressible turbulent boundary layer when the Prandtl number is not unity and the highly accelerating flow is expected on a surface.[3]

In this study, the boundary-layer integral method is applied to the thermal analysis of a jet vane, which is one type of thrust-vector control system. Jet vanes are used on small solid rocket motors due to advantages such as low actuation torque, small installation envelope, and fast-response capability.[4] However, during the operation, it experiences heavy thermal load due to direct exposure to hot exhaust gases. Also, due to the reaction with the combusted gases, it is ablated chemically and mechanically.

For the thermal analysis of vanes, potential flow around a jet vane is calculated with a CFD solver and the result is used as known values to solve the integral form of thermal boundary-layer equations. From the derived thermal boundary conditions, transient conduction in a jet vane is analyzed to predict the heating rate to the vane actuating part. Finally, the temperature history of the heated vane material is compared to experimental results from the verification of an applied method.

II. Calculation Model and Method

In the present study, FLUENT vs 6.1.22 is used to simulate the potential flow around a jet vane and the heat conduction in it. A user subroutine is inserted in the FLUENT solver to calculate the thermal boundary conditions on the vane surface. The thermal boundary layer integral method is embodied in this user subroutine.

A. Thermal Boundary Layer Integral Method

For the boundary layer analysis, the flow field near a jet vane surface is divided into several regions according to flow characteristics and the proper relations are applied to each region. Most of the applied relations, such as integral energy equations in a boundary layer, local Stanton number, and recovery temperature relations are referred to or derived from the relations suggested by Kays and Crowford[5] Also, a detailed explanation for a division of vane surface or applied relations is presented well in previous papers.[6,7]

Figure 1 shows a cross-section of a jet vane and represents the divided characteristic regions. The flow near a stagnation point of a vane nose shows a motion similar to that on a stagnation point of a cylinder whose radius of curvature is the same to that of the vane nose. Therefore, the thermal boundary conditions for the vane stagnation point can be taken as follows:

$$St_R = 0.8061 Re_R^{-0.5}\, Pr^{-0.6}, \qquad T_r = T_0 \tag{1}$$

The Stanton number on the transient sonic point is calculated solving the following two equations for thermal boundary layer and

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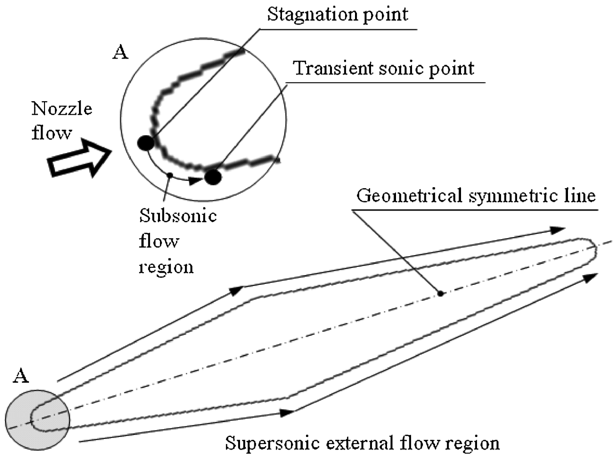


Fig. 1 Schematic diagram of a jet vane cross section.

local Stanton number:

$$\frac{q_0''}{c_p} = \frac{\mathrm{d}}{\mathrm{d}x}[\Delta_2 \rho_\infty u_\infty (T_r - T_w)] \tag{2}$$

where

$$\Delta_2 = \frac{\int_0^\infty \rho u i_s \,\mathrm{d}y}{\rho_\infty u_\infty i_{s,0}}, \qquad St = 0.0125 Pr^{-0.5} \left(\frac{T_w}{T_r}\right)^{-0.3125} Re_{\Delta_2}^{-0.25}$$

Recovery temperature at this point is

$$T_r = T_0 \frac{1 + r(k-1)/2}{(k+1)/2} \tag{3}$$

where $r = Pr^{0.33}$.

The subsonic region near a vane nose is much smaller than the supersonic region on the lateral surface. Therefore, heat-transfer coefficients and recovery temperatures in this region are calculated by interpolating from values on the stagnation point and the transient sonic point.

Supersonic flow moves along a vane lateral surface, and a turbulent boundary layer is developed on the surface. In previous studies,[1,3,5] it has been reported that the boundary-layer integral equation in a turbulent flow with constant freestream velocity can give an approximate solution even for flow with nonconstant freestream velocity such as that on the vane lateral surface. Therefore, the local Stanton number in Eq. (2) can be used again, but the flow compressibility effect and the temperature difference between the fluid and the vane surface should be corrected as follows:

$$St = 0.0125 Pr^{-0.5} Re_{\Delta_2}^{-0.25} f(T_w, M_1) \tag{4}$$

where

$$f(T_w, M_1) = (T_w/T_r)^{-0.35}(1 + r[(k-1)/2]M_1{}^2)^{-0.55}$$

For the calculation of enthalpy thickness in Eq. (4), Eqs. (2) and (4) are coupled and the following integral equation is calculated along the vane lateral surface:

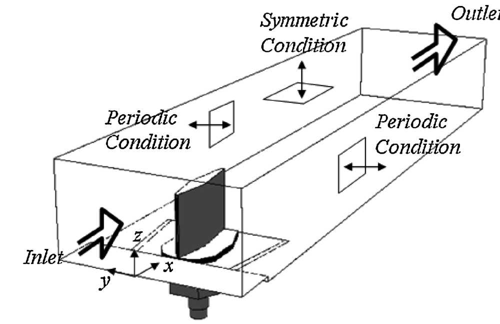
$$[\rho_\infty u_\infty \Delta_2 (T_0 - T_\infty)]^{1.25} = [\rho_\infty u_\infty \Delta_2 (T_r - T_w)]^{1.25}_{x0}$$
$$+ 0.015625 Pr^{-0.5} \int_{\alpha_0}^{\infty} f \frac{(\rho_\infty u_\infty (T_r - T_w))^{1.25}}{(\rho_\infty u_\infty / \mu_\infty)^{0.25}} \,\mathrm{d}x \tag{5}$$

Recovery temperature in the supersonic region is calculated from the following equation:

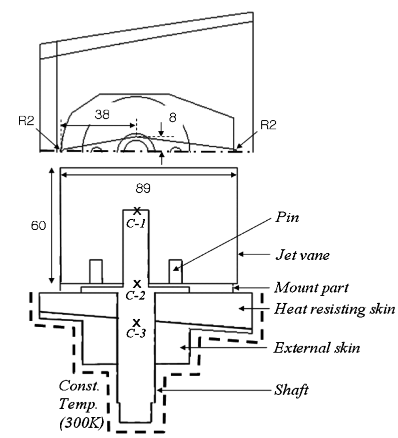
$$T_r = T_0 \frac{1 + r[(k-1)/2]M_1^2}{1 + [(k-1)/2]M_1^2} \tag{6}$$

B. Calculation Domain, Conditions, and Procedure

Figure 2 shows the entire calculation domain and the detailed shape of vane system. For simplification, a single jet vane is

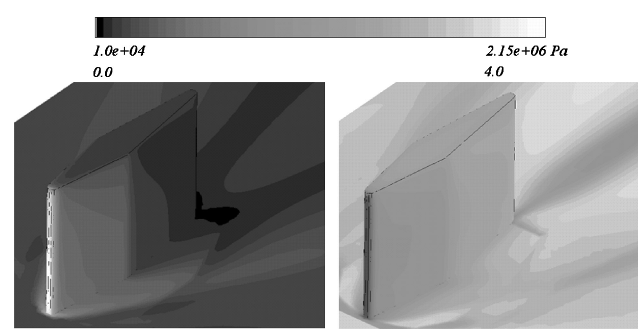


a) Entire domain



b) Shape of a jet vane system

Fig. 2 Calculation domain.



a) Static pressure **b) Mach number**

Fig. 3 Flow field around a jet vane (vane surface and plane at $z = 0$ mm).

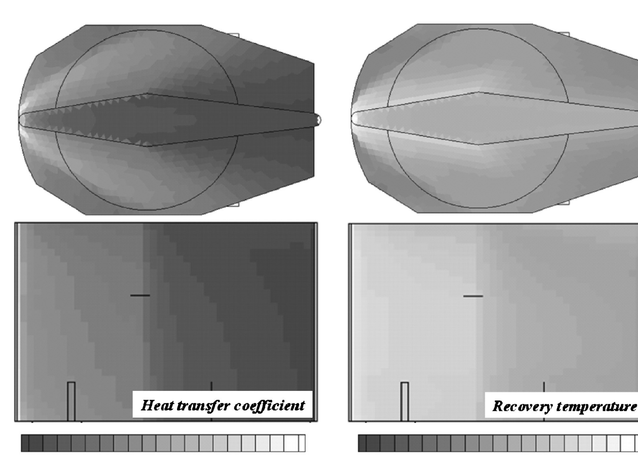


Fig. 4 Thermal boundary conditions for the conduction analysis of a jet vane system.

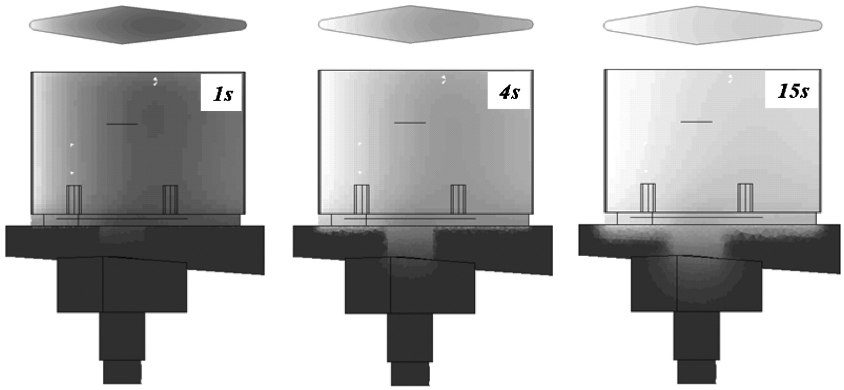


Fig. 5 Temperature distribution in a jet vane (planes at $z = 30$ mm and $y = 0$ mm).

positioned in a rectangular duct, not in a cylindrical nozzle. The periodic and symmetric conditions are set to the side boundary surfaces and the upper boundary surface, respectively. For realistic inlet conditions, another flow analysis has been performed about the supersonic nozzle, which is located in front of the jet vane system.[7] In the inlet section, averaged values of total and static pressure are 12.13 and 0.14 MPa, respectively. Total temperature values are fixed to 3000 K as inlet condition and the ambient pressure condition given to the outlet surface. In a fluid zone, the total number of numerical cells is about 786,000 and this is confirmed to give grid-independent results through the various grid tests. The relative fine meshes are given around a jet vane system for the acquisition of detailed flow properties on a surface. However, there is no strict limitation of mesh size for accurate calculation on a solid surface because the analytic method is used for the consideration of the boundary layer.

Figure 2b shows the detailed vane-system geometry and some dimensions. It consists of vane, mount block, heat-resisting skin, pin, and shaft. These elements are made of different materials, such as copper infiltrated tungsten, tungsten/zirconium/molybdenum, silica/phenolic, and steel. In this study, the correct physical and thermal properties for different materials are assumed, including their changes with temperature. Initial temperature of the solid zone is assumed to be uniform with 293 K. The total number of numerical cells is about 210,000.

III. Results and Discussions

Figure 3 shows static pressure and Mach number distributions around a jet vane. The detached bow shock wave appears before the vane and the expansion fan can also be observed at the vane edge at the middle chord, where the surface incident angle changes abruptly. Among these calculated flow properties around a jet vane system, those of cells adjacent to the vane surface are used as input values for the thermal boundary-layer integral method.

The thermal boundary conditions for vane conduction analysis are shown in Fig. 4. Both heat-transfer coefficient and recovery temperature have high values on a leading edge and they decrease gradually to the trailing edge along the lateral surface. The decrease of heat transfer in a chord direction on a lateral surface can be explained by the development of a thermal boundary layer and the increase of flow boundary-layer thickness. Also, the lower lateral surface shows a higher heat transfer coefficient and it seems to be related to the higher compressible effect on it. The similarity between two distributions of pressure (Fig. 3a) and heat-transfer coefficient (Fig. 4) can make this explanation clear. The recovery-temperature distribution is comparable with the Mach number distribution on a vane lateral surface in Fig. 3b because the recovery-temperature changes depend only on the Mach number in Eqs. (3) and (6).

A transient conduction solution in a jet vane is presented in Fig. 5. The physical time for heating of a jet vane is limited to 15 s, which is the general jet vane operation time, and the initial temperature of a jet vane system is set uniformly to 293 K. In Fig. 5, the vane nose part is heated very quickly due to high values of the heat-transfer coefficient and the recovery temperature. Due to the low conductivity of a heat-resisting material ($k_{\mathrm{Si/Ph}} = 2$–$3\ \mathrm{W/m \cdot K}$), which covers the

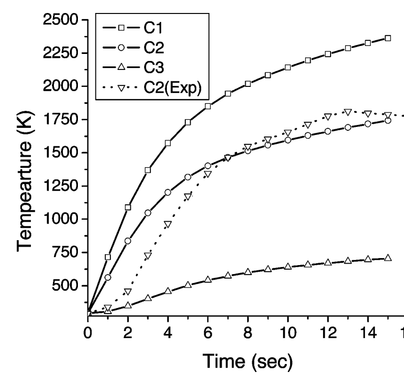


Fig. 6 Temperature history in a jet vane.

nozzle external skin (see Fig. 2b), the heat is not conducted easily to the nozzle external skin and a small amount is transferred only through the vane shaft. For quantitative comparison, the temperature history at several points in a jet vane is plotted in Fig. 6. The referred positions are presented as cross marks in Fig. 2b. The experimental result that has been obtained under similar conditions is shown for comparison as dotted line in the figure. As shown in Fig. 6, the temperature of a position C-1 increases to about 2350 K in 15 s and the position C-2 is also heated very quickly up to about 1750 K, although it is far from the vane surface. Position C-3 is protected by the heat resisting material and heated a little. This result is comparable with the experiment's result and the temperature levels are similar to each other. However, in the initial heating moment, the analysis result does not follow the experiment's result and it seems to be related to the flow transition in a nozzle just after the rocket launching.

IV. Conclusions

The thermal analysis of a jet vane system has been conducted by the hybrid method of CFD and the boundary layer analysis. Using a CFD solver, the potential flow around a jet vane and the transient heat conduction in it are simulated while the heat flux on a vane surface is calculated with the analytic method for the thermal boundary layer. The analytic method for the thermal boundary layer has been embodied in user-defined code, which shows good integration with the CFD solver. From the results, it is observed that the vane nose part is heated quickly and the thermal protection effect of heat-resisting skin is also confirmed. The temperature history in the analysis result shows good agreement with the experimental result. Hereafter, if the empirical corrections were applied to some local regions where the analytic approach is difficult, the boundary-layer integral method could be used as an effective and economical method for the thermal design of a jet vane system.

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